Experiment 1: Density Determinations and Various Methods to Measure Volume

GOAL AND OVERVIEW

This lab provides an introduction to the concept and applications of density measurements. The densities of brass and aluminum will be calculated from mass and volume measurements. The composition of a mixed brass-aluminum cylinder and the volume of empty space within a hollow cylinder will also be found. To illustrate the effects of precision on data, volumes will be determined by three different methods: geometrically (measuring lengths); water displacement; and pycnometry.

Objectives of the data analysis:

- Determine volume by three different methods
- Use measured volumes and masses to calculate densities
- Use the relationship between mass, volume, and density to find desired unknown quantities
- Evaluate results using error analysis

SUGGESTED REVIEW AND EXTERNAL READING

- Data analysis introduction (pp. 13-19), textbook information on density

INTRODUCTION TO DENSITY

The density, ρ, of an object is defined as the ratio of its mass to its volume. When combined with other distinguishing characteristics (such as color, physical state, melting point, boiling point, etc.), density is useful in identifying substances. It is also a convenient property because it provides a link (or conversion factor) between the mass and the volume of a substance.

$$\rho = {}^{m}\!/_{V}$$

Mass and volume are extensive (or extrinsic) properties of matter – they depend on amount. Density, an intensive (or intrinsic) property, is a kind of "heaviness" factor. In macroscopic terms, density reflects how much mass is packed into a given three-dimensional space. Typically, densities are reported g/ml or g/cm^3 (which are equivalent units because 1ml ≡ 1cm^3). Experimentally, mass and volume measurements are required to calculate density.

Masses are measured on electronic balances which oppose the downward gravitational attraction of an object and display the force necessary to support the object on the pan. Pan balances, which are accurate to ±0.01 g, are used for quick measurements where greater precision is not required. Analytical balances (accurate to ±0.0001 g) are used for more precise measurements.

Volume is an amount of space, in three dimensions, that a sample of matter occupies. The number and the phase of the molecules in the sample primarily determine the volume of a substance. Volume will be measured in many ways in this course, but the units are usually milliliters (mL) or cubic centimeters (cm^3). Methods for determining or delivering precise volumes include volumetric pipets and pycnometers; less precise methods include burets, graduated cylinders, and graduated pipets.

In this experiment, you will determine densities of four different metal cylinders. In parts 1–3, three different methods are used to find volume of two solid metal cylinders (Al and brass). Each method has its own degree of precision. In parts 4–5, one method for volume determination is used to find:

(i) the volume of a void inside a hollow cylinder; and,
(ii) the percent composition of a mixed-metal cylinder.

A cylinder is a standard geometric form that has an equation describing its volume. In this case, you can measure the dimensions of the cylinder and apply the formula to calculate its volume.

(1) $$V = \pi\left(\frac{d}{2}\right)^2 l = \frac{\pi}{4} d^2 l$$ where d = diameter and l = length.

For less defined shapes, volume can be determined by water displacement. Volumes of liquids such as water can be readily measured in a graduated cylinder.

To use the water displacement method, an object is inserted into a graduated cylinder partially filled with water. The object's volume occupies space, displacing liquid and raising the water level. The difference between the two volumes, before and after the object was inserted, is the object's volume.

(2) $$V_{cyl} = V_{final} - V_{initial} = V_{water+cyl} - V_{water}$$

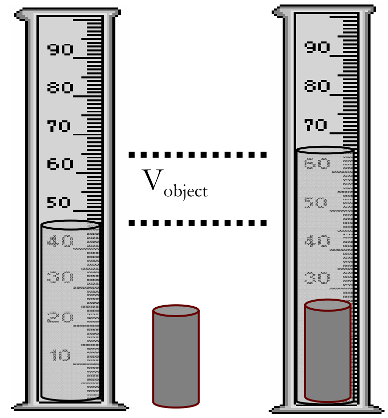


Fig. 1

Water displacement by a submerged solid can be used in a slightly different and potentially more precise way to find the solid's volume. Pycnometry is a technique that uses the density relationship between volume and mass, and the vessel used is called a *pycnometer*

To perform pycnometry measurements, the mass of the object and the mass of a flask filled with water to a mark (A, Fig. 2) are recorded. The object is then inserted into the flask.

The volume of water displaced by the object's volume is removed by pipet, thereby restoring the water level to the mark (B). The combined mass the flask, remaining water, and object is then measured.

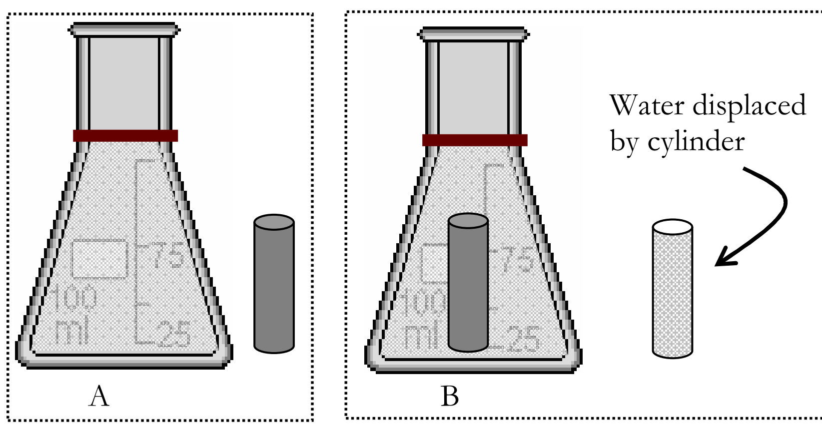


Fig. 2

(3) $$mass_A + mass_{object} = mass_B + mass_{displaced\ water}$$
$$mass_{displaced\ water} = mass_A + mass_{object} - mass_B$$
$$V_{displaced\ water} = V_{cylinder} = mass_{displaced\ water} / density_{water}$$

In B, the object's mass has been added and the displaced water's mass has been removed. You can calculate the mass of the displaced water using Eq. 3 and the volume of the displaced water using the density equation: $V = m/\rho$. The object's volume is equal to the displaced water's volume.

PRELAB HOMEWORK *(to be filled out in your bound lab notebook before you perform the experiment)*
Title and date
Define: (1) density
Answer:

1. How is density related to volume and mass? Write the equation.
2. What is the formula for the area of a circle, in terms of its diameter?
3. If 55% of an object is nickel, what fraction (in decimal format as in next part of question) of the object is nickel? If 0.045 is the fraction of an object that is lead, what percent of it is lead?

Procedure (Experimental Plan)
Try an outline or flowchart that links measured quantities with derived values and with concepts.
Data tables

EXPERIMENTAL
Equipment list:

- cylinders: brass, aluminum, mixed brass/aluminum, and hollow
- Vernier caliper
- 50 mL Erlenmeyer flask, 100 mL graduated cylinder, 400 mL beaker
- lab marker
- Pasteur pipet
- thermometer

PROCEDURE

NOTE: IF YOU WORK WITH ANOTHER SET OF PARTNERS, MAKE SURE YOU RECORD ALL DATA. YOU WILL NOT BE ABLE TO COMPLETE THE DATA ANALYSIS IF YOUR DATA TABLES ARE INCOMPLETE. ALSO CHECK THAT DATA MAKES SENSE.

Parts 1-3. Density of aluminum and brass cylinders using *three different methods of volume measurement*

<u>Part 0.</u> Measure metal cylinder masses.

1) Obtain four cylinders — brass, aluminum (solid cylinders marked S), hollow (marked H), mixed brass/aluminum (marked P for "plugged). Return cylinders to the stockroom at the end of lab.
2) Record the cylinders' numbers.
3) Record the masses of the cylinders on the analytical balance. You will use these masses throughout the experiment.

<u>Part 1.</u> Volume by Geometry

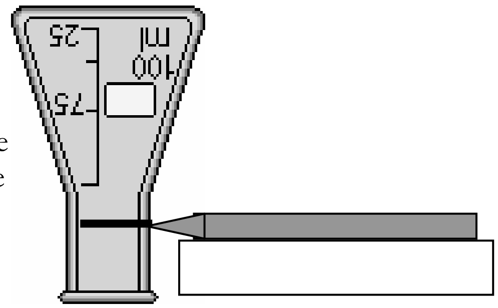
1) Measure the diameter and length of each cylinder using the Vernier calipers. Your TA will demonstrate.
2) Record the dimensions of the cylinders and the uncertainties in your readings. Be as *precise* as possible.

<u>Part 2.</u> Volume by Displacement

1) Put enough water to cover the metal cylinder into a graduated cylinder and record the volume.
2) ***Carefully*** slide the metal cylinder down the side of the graduated cylinder into the water. Tossing it in could break out the bottom of the graduated cylinder.
3) With the metal cylinder completely submerged, record the new volume reading.
4) Write down the uncertainty in your volume measurements.

Part 3. Volume by Pycnometry

1) Fill a 400 mL beaker with water and *measure its temperature*. Use this water throughout the experiment.
2) Make your pycnometer.
   a. Draw a ring midway up the neck of a 50 mL Erlenmeyer flask with a waterproof marker or wax crayon, as shown below.
   b. Invert the flask on the table; hold marker on top of something solid; and, rotate the flask while marking the neck at a constant height.



3) Calibrate your pycnometer. How well can you adjust the water's meniscus to the *top* of the line drawn? Precise filling to that mark increases reproducibility (and data quality). Practice with the pycnometer before making measurements. Your TA will demonstrate.
   a. Use a disposable pipet to add and remove drops of water to adjust the meniscus to the top of the line.
   b. Record the mass of the flask and water. Be sure there are no drops on the neck of the flask above the water line.
   c. Take out a bit of water and again fill to the mark; reweigh.
   d. Repeat a third time. These repeated mass measurements will be used to determine the uncertainty associated with your pycnometry data.

4) Indirectly **measure the mass of water displaced by your solid cylinders**.
   a. Carefully insert a metal cylinder, fill with water to the mark, and record the mass (the flask with water and cylinder).
   b. Repeat filling and weighing several times until the data appears reproducible.
   c. Repeat with the other solid cylinder.

Please do not throw the metal cylinders away – return them to the stockroom.
Put disposable pipets and broken glassware in broken glass containers, not the trash can.

Part 4. Determine Void Volume in a Hollow Cylinder by Pycnometry

1) For the hollow cylinder, record type of metal of the hollow cylinder (either aluminum or brass). You recorded its mass at the beginning of the experiment.
2) Insert the cylinder into the pycnometer; remove the water above the line, and record the new mass.

Part 5. Mass Fraction of Al and Brass Determination for Mixed-metal Cylinder by Pycnometry

1) You recorded its mass of the mixed metal cylinder at the beginning of the experiment.
2) Insert the mixed Al/brass cylinder into the pycnometer, remove the water above the line, and record the mass.

DATA ANALYSIS

Parts 1-3: Density of solid brass and aluminum cylinders

Masses were determined on the analytical balance for the two solid cylinders. Use the density equation to determine density once you know the volume (as found by each method).

$$\rho_{solid\ cylinders} \equiv \frac{m_{analytical\ balance}}{V_{various\ methods}}$$

Part 1. Determination of density using *geometry* to determine the cylinder's volume

1) The volume, V, of a cylinder is the product of its length L, multiplied by its cross-sectional area, A:
2) Find the densities of the aluminum and brass cylinders (masses divided by volumes).
3) Calculate the uncertainty in your measurements by error propagation.

$$V_{geometry} = A \cdot L = \frac{\pi d^2 L}{4}$$

Part 2. Density determination using water *displacement* to determine the cylinder's volume

1) Find the volume of each metal cylinder by subtracting the two water volumes.
2) Calculate each cylinder's density using volume and mass.
3) Calculate the uncertainty in your data by error propagation.

Part 3. Density determination using *pycnometry* to determine the cylinder's volume

1) Examine the pycnometry figure and note that material is just moving to different locations:

$$mass_A + mass_{object} = mass_B + mass_{displaced\ water}$$

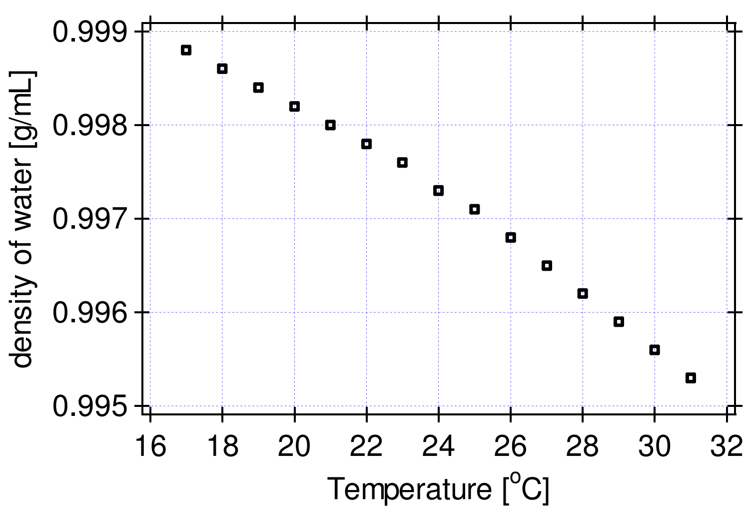
$$mass_{displaced\ water} = mass_A + mass_{object} - mass_B$$

$$V_{displaced\ water} = V_{cylinder} = {mass_{displaced\ water}}/{density_{water}}$$

You have measured the first three quantities in this equation. Solve the equation for the mass of displaced water.

2) Calculate the volume of displaced water. Use the density of water appropriate to the temperature and the displaced water's mass. You can look up water's density online (use a search).

This is the *volume of your metal cylinder, the object that displaced this amount of water.*



Density of Water as Function of Temperature

3) You have found the mass of A several times (the mass of the flask plus the water that just fills the flask). Calculate the average mass of A and the standard deviation. Let this standard deviation represent the uncertainty in all mass measurements made using the pycnometer (m_A, m_B).
4) Now that you know the volume of the cylinder, calculate its density. Propagate the errors into an uncertainty in the density if required.

Part 4. Determination of the void volume in a hollow cylinder using pycnometry

1) Determine the volume of the hollow cylinder using the pycnometer (metal plus empty space).
2) You need to calculate the volume occupied by just the metal portion of the cylinder by using the mass and the density of either aluminum or brass ($V_{metal} = mass_{metal, balance} \div \rho_{metal, pycnometry}$).
   The difference between metal volume and the actual cylinder volume (by pycnometry) is the volume of the empty space ($V_{void} = V_{cylinder} - V_{metal}$).
3) Perform error propagation to find the uncertainty in the void volume if requested.

Part 5. % composition determination of aluminum and brass in a mixed cylinder using pycnometry

1) You have measured the mass, m_{cyl}, and volume, V_{cyl}, of the mixed cylinder. You have also measured the densities, ρ_{Al} and ρ_{brass}, of aluminum and brass.

2) You want to calculate the fraction, X, of the cylinder's mass that is aluminum and the fraction, $(1 - X)$, of its mass that is brass.

   The total mass, m_{cyl}, is made up of $m_{Al} + m_{brass}$, which are Xm_{cyl} and $(1 - X)m_{cyl}$, respectively.

   The total cylinder volume is the sum of the volumes of the two metals: $V_{cyl} = V_{Al} + V_{brass}$

   Replace each volume by its mass divided by its density using $V=m/\rho$: $$V_{cyl} = \frac{m_{Al}}{\rho_{Al}} + \frac{m_{brass}}{\rho_{brass}}$$

   Replace the masses by the equivalent expressions in terms of X and m_{cyl}: $$V_{cyl} = \frac{X\, m_{cyl}}{\rho_{Al}} + \frac{(1 - X) m_{cyl}}{\rho_{brass}}$$

   Divide through by m_{cyl} and replace V_{cyl}/m_{cyl} with $1/\rho_{cyl}$: $$\frac{1}{\rho_{cyl}} = \frac{X}{\rho_{Al}} + \frac{(1 - X)}{\rho_{brass}}$$

   Collect terms on the right-hand side that contain X: $$\frac{1}{\rho_{cyl}} = X\left(\frac{1}{\rho_{Al}} - \frac{1}{\rho_{brass}}\right) + \frac{1}{\rho_{brass}}$$

   Solve for ***X*****, the mass fraction of aluminum** in the mixed cylinder. $$X = \frac{\left(\frac{1}{\rho_{cyl}} - \frac{1}{\rho_{brass}}\right)}{\left(\frac{1}{\rho_{Al}} - \frac{1}{\rho_{brass}}\right)}$$

3) When finding *X:*
   a. Calculate each fraction in the equation, then the differences, and then the final ratio.
   b. Use the densities of brass and aluminum determined experimentally.
   c. Also use X to determine the **mass fraction of brass** in the mixed cylinder, 1-X.
      X has a range of possible values from zero to one (0 – 100 %). If your mixed cylinder's density is between that of aluminum and of brass, you should calculate a percent of aluminum that makes sense. For example, if the mixed cylinder has a density near that of Al, X should be near one.

REPORTING RESULTS - Complete your lab summary

If a report is required in place of a lab summary

Abstract

Results/Sample Calculations:

masses and volumes by each method

volume of void determination

mass fraction determination

error analysis for parts 1-4

Discussion:

what you found out (refer to results tables) and how for all 5 parts

compare the three methods used to determine volume

Which method was more accurate and why?

What could be done to improve the accuracy in any or all of the methods?

How does the instrument error compare to standard deviation error?

What can you conclude from this study?

Review Questions

REVIEW QUESTIONS

1. If your volume measurement is only good to three significant figures, will your results be more precise if you use mass measurements that have more than three significant figures?
2. In pycnometry, what physical quantity (mass? volume? temperature? etc.) do you measure to determine the volume of an object?
3. List the three methods of volume measurements in order of decreasing precision. Explain why the precision decreases with each method.
4. If X is the fraction of copper in a copper-silver alloy, what is the percent silver when $X = 0.23$?