



- (c)  $\text{MX}_3$ , trigonal planar; (d)  $\text{MX}_3$ , pyramidal; (e)  $\text{MX}_3$ , T-shaped (as in  $\text{ClF}_3$ ); (f)  $\text{MX}_4$ , tetrahedral; (g)  $\text{MX}_4$ , square planar; (h)  $\text{MX}_4$ , irregular tetrahedral (as in  $\text{SbF}_4^-$ ); (i)  $\text{MX}_5$ , square pyramidal; (j)  $\text{MX}_5$ , trigonal bipyramidal; (k)  $\text{MX}_6$ , octahedral.
- 2.4 Using the  $C_{3v}$  multiplication table (Table 2.5), verify that the three  $\sigma_v$  planes belong to the same class.
- 2.5 In  $C_{3v}$  both  $C_3$  and  $C_3^2$  belong to the same class, listed as  $2C_3$  in the character table. As members of the same class their characters for any representation are the same.
- (a) Demonstrate that both  $C_3$  and  $C_3^2$  have a character of 1 for the  $A_1$  representation, by which  $z$  transforms.
- (b) Demonstrate that both  $C_3$  and  $C_3^2$  have a character of  $-1$  for the  $E$  representation, by which  $x$  and  $y$  transform degenerately. [Hint: Write the  $2 \times 2$  transformation matrices describing the actions of the operations on a point  $(x, y)$  and determine their characters.]
- 2.6 Describe the implied symmetry of the following irreducible representations on the basis of their Mulliken symbols: (a)  $A_g$  in  $C_{2h}$ , (b)  $B_2$  in  $C_{4v}$ , (c)  $E$  in  $D_3$ , (d)  $A_1''$  in  $D_{3h}$ , (e)  $E'$  in  $D_{3h}$ , (f)  $B_{1g}$  in  $D_{4h}$ , (g)  $E_u$  in  $D_{4h}$ , (h)  $T_g$  in  $T_h$ .
- 2.7 Construct real-number representations by combining the complex-conjugate paired irreducible representations in the following point groups: (a)  $C_4$ , (b)  $C_6$ , (c)  $C_5$ , (d)  $C_7$ .
- 2.8 Fill in the missing characters in the character table below, which is presented in standard format. The symbols  $A$ ,  $B$ ,  $C$ , and  $D$  represent certain symmetry operations, and  $E$  is identity.

	$E$	$2A$	$B$	$2C$	$2D$
$\Gamma_1$					
$\Gamma_2$	1		1	$-1$	$-1$
$\Gamma_3$	1	$-1$	1	1	$-1$
$\Gamma_4$	1	$-1$	1		1
$\Gamma_5$		0		0	0

- 2.9 For the point group  $O$  (cf. Appendix A), show that the group conforms to the five generalizations from the Great Orthogonality Theorem presented in Section 2.5.
- 2.10 Consider the point group  $C_3$ , consisting of the operations  $E$ ,  $C_3$ , and  $C_3^2$ .
- (a) Write the three transformation matrices for a general vector  $\mathbf{v}$  under the operations of the group, thereby forming a reducible representation  $\Gamma_{\mathbf{v}}$ .
- (b) By block diagonalization, reduce  $\Gamma_{\mathbf{v}}$  into two representations,  $\Gamma_z$  and  $\Gamma_{x,y}$ , by which the unit vector  $\mathbf{z}$  and the pair of unit vectors  $\mathbf{x}$  and  $\mathbf{y}$  transform, respectively.
- (c) Rewrite  $\Gamma_{x,y}$  as a representation of characters, and show that it is equivalent to the complex-conjugate pair of irreducible representations designated  $E$  in the  $C_3$  character table.
- (d) Explain why the representation  $E$  in the group  $C_3$  must be a pair of complex-conjugate irreducible representations.